
Quaternions

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Zach Chartrand

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INSTALLATION

Install with pip:

```
pip install quaternions-for-python
```

If you want to build from source, clone the GitHub repository:

```
git clone https://github.com/zachartrand/Quaternions.git
```


SUPPORT

The best way to help this project is to open an issue on Github.

Github: <https://github.com/zachartrand/Quaternions/issues>

OVERVIEW

3.1 The Quaternion Class

The main aspect of Quaternions for Python is the Quaternion class. The best way to use it is to import it like so:

```
>>> from quaternions import Quaternion
```

To create a quaternion, simply type

```
>>> Quaternion(a, b, c, d)
```

where a , b , c , and d correspond to a quaternion of the form $a + bi + cj + dk$. For example, creating the quaternion $1 - 2i - 3j + 4k$ looks like this in the Python interpreter:

```
>>> q1 = Quaternion(1, -2, -3, 4)
>>> q1
Quaternion(1.0, -2.0, -3.0, 4.0)
>>> print(q1)
(1 - 2i - 3j + 4k)
```

Quaternions have mathematical functionality built in. Adding or multiplying two quaternions together uses the same syntax as ints and floats:

```
>>> q1, q2 = Quaternion(1, -2, -3, 4), Quaternion(1, 4, -3, -2)
>>> print(q1)
(1 - 2i - 3j + 4k)
>>> print(q2)
(1 + 4i - 3j - 2k)
>>> print(q1 + q2)
(2 + 2i - 6j + 2k)
>>> print(q1 - q2)
(-6i + 0j + 6k)
>>> print(q2 - q1)
(6i + 0j - 6k)
>>> print(q1 * q2)
(8 + 20i + 6j + 20k)
>>> print(q2 * q1)
(8 - 16i - 18j - 16k)
>>> print(q1/q2)
(-0.19999999999999996 - 0.8i - 0.4j - 0.4k)
>>> print(1/q2 * q1)
```

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```
(-0.19999999999999996 + 0.4i + 0.4j + 0.8k)
>>> print(q2/q1)
(-0.19999999999999996 + 0.8i + 0.4j + 0.4k)
```

3.2 The qmath module

The qmath module functions similarly to Python's built-in `cmath` module for complex numbers, allowing mathematical functions to be compatible with quaternions. Here are a few examples:

```
>>> from quaternions import Quaternion, qmath
>>>
>>> q = Quaternion(1, -2, -3, 4)
>>> print(q)
(1 - 2i - 3j + 4k)
>>>
>>> print(qmath.exp(q))
(1.6939227236832994 + 0.7895596245415588i + 1.1843394368123383j - 1.5791192490831176k)
>>>
>>> print(qmath.log(q))
(1.7005986908310777 - 0.5151902926640851i - 0.7727854389961277j + 1.0303805853281702k)
>>>
>>> print(qmath.sqrt(q))
(1.7996146219471076 - 0.5556745248702426i - 0.833511787305364j + 1.1113490497404852k)
```

REFERENCE

Main reference for the Quaternions API.

4.1 Quaternions

4.1.1 The Quaternion class

```
class quaternions.Quaternion(real: float = 0.0, i_component: float = 0.0, j_component: float = 0.0,  
                             k_component: float = 0.0)
```

Quaternions are an expansion of the complex numbers, where there are four (4) components—the real component, also known as the scalar part, and the imaginary components, which together are known as the vector part. The vector part is made up of three (3) components whose unit values are i , j , and k . The rules for these values are as follows:

$$i^2 = j^2 = k^2 = -1$$

$$jk = -kj = i$$

$$ki = -ik = j$$

$$ij = -ji = k,$$

which leads to the following statement:

$$ijk = -1.$$

The descriptions will reference a quaternion of the form $a + bi + cj + dk$, where a , b , c , and d are real numbers.

Parameters

- **real** – The real component (a) of the quaternion.
- **i_component** – The i component (b) of the quaternion.
- **j_component** – The j component (c) of the quaternion.
- **k_component** – The k component (d) of the quaternion.

Each component can be returned by calling the attribute of the same name.

Example

```
>>> q = Quaternion(1, -2, -3, 4)
>>> print(q)
(1 - 2i - 3j + 4k)
>>> q.real
1.0
>>> q.i
-2.0
>>> q.j
-3.0
>>> q.k
4.0
```

4.1.2 Methods

Normal methods

`Quaternion.conjugate()` → *quaternions.quaternions.Quaternion*

Return the conjugate of self. This is analogous to the complex conjugate, reversing the signs of the vector components.

`Quaternion.get_imag()` → *float*

Return the imaginary component of the quaternion if only one of the imaginary components is nonzero. If the quaternion is scalar, return `0.0`. Otherwise, return `None`.

`Quaternion.get_vector_components()` → *Tuple[float]*

Return the vector components of the Quaternion as a tuple formatted as (i, j, k).

`Quaternion.inverse()` → *quaternions.quaternions.Quaternion*

Return `1/self`.

Return the inverse of the quaternion. The inverse of a quaternion is defined as the conjugate divided by the norm squared:

```
q.inverse() = q.conjugate()/(q.norm)**2
```

`Quaternion.unit_quaternion()` → *quaternions.quaternions.Quaternion*

Return the quaternion normalized to magnitude one (1).

If the quaternion is a zero (0) quaternion, return the zero quaternion.

`Quaternion.unit_vector()` → *quaternions.quaternions.Quaternion*

Return the vector part of the quaternion normalized to a magnitude of one (1). Return the zero quaternion if the magnitude of the quaternion is zero (0).

Boolean methods

`Quaternion.is_complex()` → bool

Return True if only one of the i , j , and k components is nonzero. Otherwise, return False.

`Quaternion.is_scalar()` → bool

Return True if the vector components all equal zero. Otherwise, return False.

`Quaternion.is_vector()` → bool

Return True if the scalar part is zero and at least one of the vector components is nonzero. Otherwise, return False.

Class methods

classmethod `Quaternion.from_angle(angle: float, vector: Iterable[float], norm: float = 1.0, degrees: bool = True)` → *quaternions.quaternions.Quaternion*

Return a quaternion from an angle and vector.

Quaternions can be expressed as $\text{norm} * (\cos(\text{theta}) + \mathbf{u} * \sin(\text{theta}))$, where \mathbf{u} is a 3D unit vector. This function takes an angle and a vector to create a quaternion. If you want a quaternion with a different norm than one (1), you can change the `norm` argument. By default, angles are entered in degrees. If you want to enter an angle in radians, set `degrees` to False.

Properties

property `Quaternion.angle:` float

The angle of the quaternion in radians.

property `Quaternion.angle_in_radians`

Same as *Quaternion.angle*

property `Quaternion.angle_in_degrees:` float

The angle of the quaternion in degrees.

property `Quaternion.components:` Tuple[float]

The components of the quaternion as a tuple in the order (real, i , j , k).

property `Quaternion.norm:` float

The norm (magnitude) of the quaternion.

property `Quaternion.scalar:` *quaternions.quaternions.Quaternion*

The real part of the quaternion.

property `Quaternion.vector:` *quaternions.quaternions.Quaternion*

The vector part of the quaternion.

property `Quaternion.vector_norm:` float

The norm of the vector part of the quaternion.

property `Quaternion.versor:` *quaternions.quaternions.Quaternion*

The quaternion normalized to a magnitude of one (1).

4.2 The qmath module

4.2.1 qmath

Similar to the built-in module `cmath`, this module has definitions of mathematical functions expanded to work with quaternions.

4.2.2 Quaternion mathematical functions

`quaternions.qmath.exp(q: quaternions.quaternions.Quaternion) → quaternions.quaternions.Quaternion`

Return the exponential of a quaternion.

`quaternions.qmath.log(q: quaternions.quaternions.Quaternion, base: float = 2.718281828459045) → quaternions.quaternions.Quaternion`

Return the logarithm of a quaternion to the given base.

If the base is not specified, returns the natural logarithm (base e) of the quaternion.

`quaternions.qmath.log10(q: quaternions.quaternions.Quaternion) → quaternions.quaternions.Quaternion`

Return the base-10 logarithm of the quaternion.

`quaternions.qmath.sqrt(q: quaternions.quaternions.Quaternion) → quaternions.quaternions.Quaternion`

Return the square root of the quaternion.

4.2.3 Rotation functions

`quaternions.qmath.rotate3d(point: Iterable[float], angle: float, axis: Iterable[float] = (0.0, 0.0, 1.0), rounding: int = -1, degrees: bool = True) → Tuple[float]`

Rotate a point around an axis.

Take a point in 3d space represented as a tuple or list of three (3) values and rotate it by an angle around a given axis vector.

Parameters

- **point** – The point to rotate. The format for the coordinates is (x, y, z).
- **angle** – The angle of rotation. By default, angle is set to be input in degrees. See the **degrees** parameter if you want to use radians instead.
- **axis** – The axis to rotate the point around. By default, this is the z-axis (0, 0, 1).
- **rounding** – The number of decimal points the result will be rounded to. Default value is -1, which does not round the end result.
- **degrees** – When set to True, this function interprets the parameter **angle** as degrees. Set this parameter to False to use angles in radians. Default is True.

For the point and axis parameters, if only one value is given, the value will be assumed to be an x-coordinate with the y- and z-coordinates equal to zero (0). If two values are given, they will be assumed to be x- and y-coordinates with the z-coordinate equal to zero (0).

`quaternions.qmath.rotate_Euler(point: Iterable[float], yaw: float, pitch: float, roll: float, x_axis: Iterable[float] = (1.0, 0.0, 0.0), z_axis: Iterable[float] = (0.0, 0.0, 1.0), degrees: bool = True) → Tuple[float]`

Rotate a given point using Euler angles.

This function uses the rotation convention of z-y'-x'', rotating yaw, then pitch, then roll.

Parameters

- **point** – The point to rotate. The format for the coordinates is (x, y, z).
- **yaw** – The angle of rotation around the z-axis.
- **pitch** – The angle of rotation around the y'-axis. The y'-axis is the y-axis after the yaw rotation has been applied.
- **roll** – The angle of rotation around the x''-axis. The x''-axis is the x-axis after both the yaw and pitch rotations.
- **x_axis** – The initial x-axis of the coordinate system that **point** belongs to. Default value is (1, 0, 0).
- **z_axis** – The initial z-axis of the coordinate system that **point** belongs to. Default value is (0, 0, 1).
- **degrees** – When set to **True**, this function interprets the parameter **angle** as degrees. Set this parameter to **False** to use angles in radians. Default is **True**.

New in version 1.1.0.

4.2.4 Other functions

`quaternions.qmath.cross_product(vector1: Iterable[float], vector2: Iterable[float]) → Tuple[float]`

Return the cross product of two vectors.

Because this uses quaternions to calculate, this only works for vectors up to three (3) dimensions.

New in version 1.1.0.

`quaternions.qmath.dot_product(vector1: Iterable[float], vector2: Iterable[float]) → float`

Return the dot product of two vectors.

Because this uses quaternions to calculate, this only works for vectors up to three (3) dimensions.

New in version 1.1.0.

4.2.5 Constants

`quaternions.qmath.pi`

The mathematical constant π , as a float.

`quaternions.qmath.tau`

The mathematical constant τ , as a float.

`quaternions.qmath.e`

The mathematical constant e , as a float.

`quaternions.qmath.inf`

Floating-point positive infinity. Equivalent to `float('inf')`.

`quaternions.qmath.infj`

Quaternion with positive infinity i part and zero for all the other parts. Equivalent to `Quaternion(0, float('inf'), 0, 0)`.

`quaternions.qmath.infj`

Quaternion with positive infinity j part and zero for all the other parts. Equivalent to `Quaternion(0, 0, float('inf'), 0)`.

`quaternions.qmath.infk`

Quaternion with positive infinity k part and zero for all the other parts. Equivalent to `Quaternion(0, 0, 0, float('inf'))`.

`quaternions.qmath.nan`

A floating-point “not a number” (NaN) value. Equivalent to `float('nan')`.

`quaternions.qmath.nani`

Quaternion with NaN i part and zero for all the other parts. Equivalent to `Quaternion(0, float('nan'), 0, 0)`.

`quaternions.qmath.nanj`

Quaternion with NaN j part and zero for all the other parts. Equivalent to `Quaternion(0, 0, float('nan'), 0)`.

`quaternions.qmath.nank`

Quaternion with NaN k part and zero for all the other parts. Equivalent to `Quaternion(0, 0, 0, float('nan'))`.

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